

mination of Sex." The peculiar position of Lepidoptera and Aves with regard to these matters is emphasised.

A welcome section on "Germ-cell Determinants" gives clearly the main facts which have been ascertained. The author is commendably cautious in his discussion of this interesting subject, and recognises that "although these bodies are evidently strictly correlated with the germ-cells, there is no absolute certainty that they are the cause of the differentiation of germ-cells from body-cells."

Most cytologists will concur with Prof. Doncaster in his view that the weight of evidence is in favour of the main theory of the individuality of the chromosomes; the author emphasises the fact that the chromosome itself is in all probability divisible into smaller units, which may have an individuality more fundamental than the chromosome as a whole, and he suggests that the individuality theory should be extended so as to regard these granules (microsomes) as the fundamental units.

The chapter on the mechanism of hereditary transmission introduces a discussion on the most recent work on *Drosophila* and *Abraxas*. Prof. Doncaster is here dealing with a subject which he has himself studied specially, and though he treats the question with impartiality, he comes to the conclusion that not only does the behaviour of the chromosomes in the maturation divisions of the germ-cells provide the mechanism required for Mendelian segregation of characters, but also that the work of Morgan on *Drosophila* carries us a step farther and gives us some idea how the groups of characters may be related to special chromosomes. The author recognises the difficulties with regard to our full acceptance of the theories of Morgan and his colleagues relating to the mechanism of "coupling" and "crossing over," but states that no other hypothesis which has been brought forward fits the main facts so well. He concludes his book with an essay on the rôle of the cytoplasm in heredity, and gives a good account of the organ-forming substances.

Arranged and written as it is, this book is certain to stir up interest in the subject of cytology. By pointing out the perfections and defects of our present-day basic cytological theories and hypotheses, the author has succeeded in emphasising the lines along which fruitful research may be followed. We hope that this book will mark the beginning of greater activity among English cytologists. Prof. Doncaster is to be congratulated warmly on this excellent work. J. B. G.

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Matrices.

University of Calcutta: Readership Lectures: Matrices and Determinoids. By Prof. C. E. Cullis. Vol. ii. Pp. xxiii + 555. (Cambridge: At the University Press, 1918.) Price 42s. net.

THE history of the mathematical term "matrix" is likely to be very interesting. Its original meaning was an array of symbols (a_{mn}) forming a rectangle of m rows and n columns, out of which determinants were selected by picking out columns (or rows) of the array. A square matrix gives only one associated determinant, but a square matrix is not the same thing as a determinant.

When we change from one set of variables to another by linear relations

$$y_j = \sum_i a_{ij} x_i \quad (i = 1, 2, \dots, n; j = 1, 2, \dots, m),$$

we have an associated matrix (a_{mn}), or A , which is square only when the number of variables is the same in each set. In practice this is the most usual case, and it will be simpler to confine ourselves to this for the present. If we take a new set of variables z_i such that

$$z_j = \sum_i b_{ji} y_i,$$

we have a matrix $B = (b_{nn})$, and by eliminating the symbols y_i we deduce

$$z_j = \sum_i c_{ij} x_i,$$

where the symbols c_{ij} are derived from A, B by a process of "composition," and form a new matrix C . We write $C = AB$ symbolically, and thus start the theory of the multiplication of matrices. There are many analogies with the theory of groups; for instance, BA must be distinguished from AB , multiplication is associative, and so on.

Cayley seems to have been the first to develop the theory of square matrices from this point of view (Phil. Trans., vols. cxlviii., clvi., and elsewhere); other English mathematicians, such as Sylvester, Buchheim, and Tait, took up the subject later on. It may be specially noted that H. Smith's memoir on linear indeterminate equations and congruences contains a great deal of the fundamental theory of matrices, both square and rectangular. In particular, there is a complete and, we believe, original statement of the existence and properties of the elementary factors of a determinant the elements of which are ordinary integers. Weierstrass, Kronecker, and Frobenius, especially the last-named, have made important contributions to the subject.

It will be seen that a matrix is now not merely a scheme of symbols used to specify a set of deter-

minants, but a kind of entity of a very abstract and comprehensive type. A large part of group-theory and many complex linear algebras can be expressed in terms of matrices, and this absorptive property of matrix-theory will probably become more evident in course of time. Matrices occupy a special section in the International Schedule, and the Royal Society Index contains about sixty titles under that heading. Papers under other headings (especially determinants) have also more or less bearing on the subject. Prof. Baker's works on Abelian functions show the importance of matrices in the general theory of theta functions. Some knowledge of the subject is becoming essential in connection with various branches of pure mathematics.

Prof. Cullis's second volume, if we understand the author aright, seems to be a continuation rather different from that which he originally planned. This is not to be regretted, because in this portion we have statements and proofs of well-known and important theorems in the author's own notation, and a large number of illustrative examples. Among the subjects treated are ranks of matrix products and factors, equigradient transformations, certain matrix equations of the second degree, and various properties of a pair of matrices ("paratomy," "orthotomy," and so on). Much of the argument is put into a quasi-geometrical form. The outstanding feature of the work, which the author properly emphasises, is the detailed discussion of rectangular, as distinguished from square, matrices. For this reason alone the work ought to give a great stimulus to the subject, and we hope that the publication of the whole treatise will not be long delayed. Until it is finished, it will be difficult, if not impossible, to give a proper appreciation of it, especially as the author introduces so many new symbols and technical terms. One thing, however, is certain: we now have the outlines of a calculus of matrices in which the operations of addition, subtraction, and multiplication are definite. It may be conjectured that some of the most important applications will be to problems connected with a compound modulus, arithmetical or algebraical as the case may be.

As a matter of curiosity it may be noted that one or two of the very first problems in the theory of rectangular matrices occur in Gauss's "*Disquisitiones Arithmeticae*"; for instance, in connection with the theory of composition of quadratic forms, we have the problem of finding a matrix ($a_{2,4}$) the six determinants of which are to be six assigned integers, subject to a certain relation.

G. B. M.

The Chemistry of Animal Products.

The Essentials of Chemical Physiology: For the Use of Students. By Prof. W. D. Halliburton. Tenth edition. Pp. xi+324. (London: Longmans, Green, and Co., 1919.) Price 7s. 6d. net.

THE fact that this well-known and appreciated text-book has reached its tenth edition is sufficient evidence that it satisfies adequately the need for a short practical course in the chemistry of the substances found in and produced by the activity of living tissues. This object is excellently attained. It is not to be expected that a detailed account of the chemical processes occurring during the life and functional action of the organs of the body is to be found therein. Indeed, it would be impossible to separate the chemical from the physical aspects of any of these physiological processes. Such a separation appears to be an unfortunate necessity in a great part of the teaching of the subject, but a more intimate union between the chemical side and what is sometimes called the "experimental" side of the student's work is very desirable, and might be arranged without much difficulty.

There are some important questions which are apt to fall out in the present arrangement; such are those of permeability, osmotic pressure, hydrogen-ion concentration, and the properties of colloidal solutions. In a future edition Prof. Halliburton might find it possible to include a few simple exercises in these problems. An intelligent grasp of the principles involved is not to be attained by the mere reading of statements about them, while even a small number of experiments have great value. On account of its importance in physiological phenomena, some experiment showing the synthetic aspect of the action of enzymes might well be inserted.

A general criticism which applies to most text-books on practical chemistry, especially to those on biological chemistry, is that a number of the tests given suggest cookery recipes rather than scientific experiments. A student is very little the better for performing Molisch's sugar test if he is ignorant of what the result is due to. And how many understand, when they make the tests, why some sugars reduce copper salts, while others do not; or why tartrates are added to Fehling's solution? It would often be better to curtail the recital of what is to be found in general text-books in order to explain the reasons for the results of the actual experiments made.

In the book before us the theoretical part is, on the whole, brought well up to date. In view of